Remarks on the History of the Probabilistic Abacus

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On November 30, 1991 I received a preprint of a paper with the title *Chip firing games on directed graphs.* It appeared later in the *Journal of Algebraic Combinatorics 1, (1992), 305-328* under the name *Chip-Firing Games on Directed Graphs.* The authors were Anders Björner, Royal Institute of Technology (Stockholm) and László Lóvasz, Eőtvős Loránd University, Budapest. Here I heard for the first time about Chip Firing Games, and the references showed me that the subject has by now become a 'hot' topic. By the way the authors give full credit to my two papers 1975/1976 in the Educational Studies in Mathematics as the first use of Chip Firing Games.

A few words about the name. In Carbondale I was engaged in the CSMP-Project. At the project in the primary grades Papy's Abacus was used by the children quite skillfully for calculations with integers and fractions. So it was quite natural for me to use the name 'Probabilistic Abacus' for a device for computing probability and expectations with the greatest ease.

Back home from Carbondale in 1974, where I discovered the Abacus, I was finishing my second volume *Einführung in die Wahrscheinlichkeitsrechnung und Statistik 2.* I squeezed in a final chapter of five pages on the Probabilistic Abacus. In their book *Charles M. Grinstead and J. Laurie Snell, Probability, American Mathematical Society, 1997* the authors referred to this book describing the content of the final chapter pertaining to Markov Chains on pages 445-447 of their AMS-Book. See Problem 29. They pose the problem of proving that the algorithm described in my book always gives the fixed vector of an ergodic chain.

I also wrote a 24-page extended translation of my first paper mentioned above for the journal *Der Mathematikunterricht,* April 1975. Then I started a lecture tour through Germany, Austria and Switzerland.

Later it turned out that the visit to Poland was of utmost importance. In summer 1975 Professor Semadeni conducted a one week conference in teaching probability and statistics for teachers and teachers of teachers. He invited me to give a talk at the Banach Center on the Probabilistic Abacus. After my talk Professor Papy furiously attacked me. It is a scandal to propose something to teachers that has not even been proved to be correct. I answered that I have the evidence of 1000 most diverse confirmations with sometimes very extensive graphs and not a single failure. Besides, proofs are the domain of professional probabilists, none of my business. In addition, the chips execute exactly my view of probability. So they must give correct results. This enraged Papy even more. Then I said: OK, I will try the proofs tonight. The task turned out to be easier than I thought. By midnight I had all the proofs, except the Critical Load Theorem (CLT). By now I was tired and went to bed. The next morning I anounced triumphantly that I succeeded with all proofs except the CLT. But the functioning of the Abacus luckily did not depend on CLT. It is just a convenience. (By the way, after my return to Frankfurt Dr. L. Scheller, then at our department, proved the CLT.)

I was urged to publish my results, best in the same Journal. Luckily Hans Freudenthal, the editor of the Journal was also present. He promised to publish the results as soon as possible. This was the origin of my second paper of 1976. By invitation I gave another lecture at the Banach Center in Warsaw to a selected group of professional statisticians with the title *Genetics by means of the Probabilistic Abacus.* Here I started to use the obvious symmetry of genetic graphs to reduce their size and thus reduce the labor considerably. I touched chips only when it was absolutely necessary. The lecture was also visited by some teachers of teachers. It showed them the enormous advantage of the Abacus on quite sophisticated problems.

Later the Abacus became quite popular in Polish schools. Professor Zawadowski from the University of Warsaw made later a lecture tour through Germany. He also came to Frankfurt and told me that in most remote places in Poland *Engelowy Graphy* are used. He even made a lecture tour through Eastern Europe, lecturing about the Abacus.

My last lecture on the Probabilistic Abacus was 1980(?). Runnenburg of the University of Amsterdam was at that time President of the Dutch Statistical Society. At its annual meeting the society invites for its only plenary session one foreign lecturer. Runnenburg invited me 1979(?) as a plenary speaker and asked me to give a talk on the Abacus. I agreed. Under optimal conditions, before an audience of 400 people it was a great success. The audience was amazed at the skill with which I moved the chips via gigantic graphs getting in 10 seconds results that are usually arrived at after extended calculations. By that time I was already pretty skillful. Especially I started to use non-obvious symmetry to reduce the graphs. So I could get very simple graphs with the same probability or expected time, but usually not both. One should mention that the Abacus is ideally suited for projections on a large screen. I used small dice as chips. They and their moves were very well visible. The Abacus gives an excellent topic for spectacular lectures.

Before my talk Runnenburg told me that he is working on the Abacus with some of his assistants. He also asked me if I could also find the variance with the Abacus. Because this is all that a statistician really needs. I said no, since probability and expectation are linear concepts, whereas variance is a quadratic concept. Now I am no more so sure.

I will now report about my last publication about the Probabilistic Abacus. Shortly before I gave my Amsterdam talk, Springer-Verlag asked me if I could design the month of August 1980(?) for their annual Mathematical Calendar. Two pages altogether, one page of text and one page for spectacular pictures. I chose the Probabilistic Abacus for my topic. It was a huge challenge for me to present on one page of text the whole theory of the Abacus. I used one nontrivial example and on this example I treated all problems that are usually done in advanced probability books. I think I was successful. Unfortunately I gave the only copy I had to my friend Lennart Råde of Chalmers Institute of Technology in Götteborg. But I still have the galley proofs.

1976 Germany decided to participate at the International Mathematical Olympiad (IMO). The first participation was scheduled for July 1977. I was urged to become Head Coach of the team. Very reluctantly I agreed, because I thought it is a good idea, but it would absorb me completely. Fortunately the mathematics in German Schools was at that time in comparatively good shape, at least in South Germany, Bavaria and Baden-Württemberg. So we could quickly attain a respectable position. By now the state of mathematical knowledge is uniformly bad except some small pockets.

By 1982 I thought I would have a position among the top 3 or 5 nations. Then I could resign and go back to my beloved Abacus. But in the summer 1982 I became gravely ill. It was one day before leaving for Hungary to the Olympiad. After three brain operations I became a different person. I started to wind up my life, and now I had different priorities. I started to write four books, which I wanted to write anyway, but considerably later after much more experience. Before my operation I needed for one book about one year. But now I needed four years. It was at least a consolation that our IMO-team took first place in the Olympiad in Hungary 1982. 1983 in Paris we took again first place, this time by a large margin.

In writing my third book *Exploring Mathematics with Your Computer*, NML-35, Mathematical Association of America, I was on the verge of introducing the Probabilistic Abacus. My editor, Peter Ungar asked me to write a short chapter on probability. After some hesitation I wrote a final Chapter of 8 pages with the title *A Crash Course in Probability*. It is completely graph oriented, but does not mention the Abacus. After completing my last and fourth book, a Springer-Volume *Problem-Solving Strategies* I have now time to come back to the Abacus. I still intend to write a school book introducing Probability with the Probabilistic Abacus. Unfortunately now the teachers are no more receptive to new ideas about teaching probability. In spite of this I will probably begin (in fact I started already) to write a book. But I am in no hurry, there are no pressures to end up at some date. I do not mind if it will never be published during my lifetime. Maybe it will be a posthumous work.

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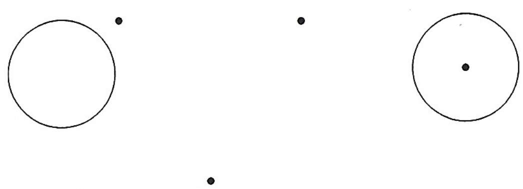
Addendum

For whom should I write a book on the abacus? The most response I got by now is from the universities worldwide. Some teach a one-semester short course and are happy to enrich their work with the abacus. So I aim primarily at those teachers teaching at the university courses for future teachers. But I do not exclude teachers teaching the grades 4 to 12. Some people wanted samples of lessons from my work in primary grades. Since I do not have the originals I attach an example from 2rd to 4th grades. The lessons at that level are mostly stories with probabilistic content. The lesson I sketch is the one I can well remember since it was used as the first one and did not yet use the Abacus.

Lesson # 1. The Thieves of Bagdad. (2nd to 4rd grade, before the abacus.)

1000 years ago there reigned in Bagdad a Khalif Harun el Rashid. He was a just and honest man. So he hated thieves. Every day he sent his police chief to the Bazar to look out for thieves. Sometimes the police chief caught one. Then he brought him to the Khalif. The thieves always asserted that they were innocent. But the Khalif put them into a dark dungeon gave them water, but no bread, and said: Run for your life! If you are innocent then Allah will save you. The dungeon was built as follows: It was round and had three tunnels: 0 , 2, 3. Tunnel 0 lead after one hour to freedom. 2 led after two days back to the dungeon. 3 led after 3 days back to the dungeon. The thieves could survive for 5 days. But at the sixth day they broke down inside 2 or 3 if they did not choose the life saving tunnel 0. The thieves knew nothing about the way the dungeon was built. Furthermore if they returned to the dungeon they did not even know that they have returned. So they were equally likely to stumble again into any of the three tunnels.

One day the Khalif recognized a thief he put into the dungeon. By that time he had already put 27 thieves into the dungeon. So he wanted to know how many were saved by the mercy of Allah. He asked his Court Mathematician (CM): How many of the thieves were saved, how many of their skeletons are in tunnel 2 and how many in tunnel 3? The CM said that he cannot tell this number, but he can give a best estimate, which is quite near to the correct numbers. He can even explain his reasoning to the Khalif. He draws Fig. 1. How many of the 27 thieves will start through the doors 0, 2, 3? The Khalif replied, anything could happen. For instance, all could go through 0 and be free, but this is unlikely. The CM continued: Let us look at the possible paths. There are 6 life saving paths: 0, 20, 30, 220, 230, 320. On the other hand there are 7 paths leading to starvation: 33, 222, 223, 232, 233, 322, 323. Since 6 of the 13 paths lead to freedom, the Khalif said one could surmise that the chances are 6:7 for freedom. Now the CM draws the tree in Fig. 3. CM: On the first stage I would say that the most likely event would be that 9 would go through each door 0, 2, 3.



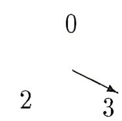


Fig. 2

(35,10)(35,0)(5,41)(10,40)(15,40)(17.5,40)(15,41)(10,50)(5,51)(2.5,46.5)(5,41) (52,40)(55,39)(60,38)(65,37)(70,40)

(10,50)(10,40) (10,50)(18.66,55)(10,50)(1.34,55)

(80,50)(40,40)(20,20)(0,0)(20,20)(20,0)(20,20)(30,0)(40,40)(40,20)(40,0) (40,20)(55,0)(40,20)(70,0)(40,40)(60,20)(80,50

In this tree the 27 thieves are distributed by splitting them equally at each vertex. We see that $9+3+3+1+1+1 = 18$ will get free, $1+1+1 = 3$ will starve in tunnel 2 and $1+1+1+3 = 6$ will starve in tunnel 3. D2 and D3 means died in 2 and died in 3, respectively. It seems that the paths have different weights, so your original conjecture, although most ingenious is not substantiated by the facts. It turns out that the paths have different weights. The shorter they are the larger their weight.

The Khalif sent his body guard to check the results found by the CM. After 3 days they came back with this result: 19 got free, 2 were found in tunnel 2 and 6 in tunnel 3, quite a good guess by the CM.

Now every child successively pushes 27 chips through the maze each controlled by the dial in Fig. 2. The results of the children are close to the predictions of the CM.

Variations of this problem:

a) The thieves always kept in touch with one wall during their way through the tunnels. In getting out they equally likely turned left or right. What are the chances of escape now?

b) The thieves knew the left hand rule of getting out of a maze: *All the time keep in touch of the wall with your left hand.* What fraction of the thieves will now escape?

Similar problems are treated using trees. Then other graphs without loops or cycles. Than a cycle is used which returns to the initial state. In these graphs no preloading is necessary. Then a loop on an interior state is used. One realizes that the game will go on forever without returning to empty interior states. Then one uses preloading. Introduction of critical preloading which always reappears. Starting with the critical load it is enough to play until the critical load reappears.

It is important that after each play one should use simulation. Each chip is run through the graph, directed by suitable random devices. Then the results are compared with the results given by the abacus.

In the CSMP-project children had huge numbers of dials equivalent to coins, 6-dice, 10-dice, etc. By suitable coloring of the sectors one also got probabilities like $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$ or $\frac{1}{4}$ and $\frac{3}{4}$. The dials have a big advantage compared to real dice or coins. Real dice or coins make noise, roll down under the desk and create a constant noise level of several decibells too much. Dials are silent, and unequal probabilities are easy to represent. One also sees the probabilities as sectors of the dial.

In 1970 a hand held calculator or computer, giving at each push of a button a new random digit was not yet existent. Today one could largely reduce the use of dials though I would not entirely eliminate them. Just in the first probability lessons I would use dials until children get a feeling for probability. Later one could say there is a small demon inside the calculator turning the dial and showing the result. The gain in speed is considerable.